# DEFENCE DREAMS

# NDA MATHEMATICS

TRIGONOMETRY • ALGEBRA • CALCULUS

Concept notes • Key formulas • Worked examples • Practice questions

## Prepared for NDA aspirants

A structured revision handbook designed for focused study. Use the examples to learn the method, then solve the practice questions without looking at the answers.

### How to use these notes

1. Learn the definitions and identities.
2. Rework every solved example on paper.
3. Attempt the practice set under a time limit.
4. Maintain a formula notebook and revise it regularly.

Note: These are original study notes aligned to common NDA Mathematics topics, not an official UPSC publication.

# How to Study Mathematics for NDA

## A practical routine

• Revise formulas for 10–15 minutes before solving questions.

• Work topic-wise first; later mix questions from different chapters.

• Show intermediate steps in practice, even when the exam only needs an option.

• After each mock test, classify errors as concept, calculation, reading, or time-management mistakes.

## A balanced session

Suggested 90-minute session: 15 minutes formula recall, 45 minutes focused problem solving, 20 minutes mixed practice, and 10 minutes error review.

## Exam technique

• Estimate the size and sign of an answer before calculating.

• Use substitution to check algebraic identities and equations.

• Do not spend too long on one difficult question; mark it and return later.

• Keep radians and degrees clearly distinguished in trigonometry and calculus.

## Prerequisites

Comfort with fractions, indices, factorisation, coordinate geometry basics, and elementary arithmetic will make these three units easier.

# 1. Angles and Measurement

## Degrees and radians

A complete revolution is 360° or 2π radians. Therefore 180° = π radians and 1° = π/180 radians.

## Conversion formulas

• Degrees to radians: θ(rad) = θ(deg) × π/180.

• Radians to degrees: θ(deg) = θ(rad) × 180/π.

• Arc length: s = rθ when θ is measured in radians.

• Sector area: A = ½r²θ when θ is measured in radians.

## Worked example

Convert 150° to radians: 150 × π/180 = 5π/6. Convert 7π/4 to degrees: (7π/4) × 180/π = 315°.

## Common angles

30° = π/6; 45° = π/4; 60° = π/3; 90° = π/2; 180° = π; 270° = 3π/2; 360° = 2π.

## Quick check

• Convert 225° to radians.

• Convert 5π/3 to degrees.

• Find the arc length for r = 6 and θ = π/3.

# 2. Trigonometric Ratios

## Right-triangle definitions

For an acute angle $\theta$: $\sin\theta$ = opposite/hypotenuse; $\cos\theta$ = adjacent/hypotenuse; $\tan\theta$ = opposite/adjacent.

## Reciprocal ratios

- $\operatorname{cosec}\theta = 1/\sin\theta$
- $\sec\theta = 1/\cos\theta$
- $\cot\theta = 1/\tan\theta$ = adjacent/opposite

## Memory aid

SOH–CAH–TOA: Sine = Opposite/Hypotenuse; Cosine = Adjacent/Hypotenuse; Tangent = Opposite/Adjacent.

## Worked example

In a right triangle, the sides relative to $\theta$ are opposite = 3, adjacent = 4, hypotenuse = 5. Then $\sin\theta = 3/5$, $\cos\theta = 4/5$, $\tan\theta = 3/4$, $\operatorname{cosec}\theta = 5/3$, $\sec\theta = 5/4$, $\cot\theta = 4/3$.

## Practice

- If $\sin\theta = 5/13$ and $\theta$ is acute, find $\cos\theta$ and $\tan\theta$.
- If $\tan\theta = 7/24$, find $\sin\theta$ and $\cos\theta$.
- State which ratios are undefined when $\cos\theta = 0$.

# 3. Standard Angles and Signs

## Standard values

θ: 0°, 30°, 45°, 60°, 90°
sin θ: 0, 1/2, √2/2, √3/2, 1
cos θ: 1, √3/2, √2/2, 1/2, 0
tan θ: 0, 1/√3, 1, √3, undefined

## Signs in quadrants

• Quadrant I: all ratios positive.

• Quadrant II: sine and cosecant positive.

• Quadrant III: tangent and cotangent positive.

• Quadrant IV: cosine and secant positive.

## Reference angles

The reference angle is the acute angle between the terminal arm and the x-axis. Use it to find the magnitude of a ratio, then apply the quadrant sign.

## Worked example

sin 150° = sin(180° − 30°) = sin 30° = 1/2. cos 150° = −cos 30° = −√3/2.

## Practice

• Evaluate cos 120°.

• Evaluate tan 225°.

• In which quadrants is cosine negative?

# 4. Fundamental Identities

## Pythagorean identities

• $\sin^2\theta + \cos^2\theta = 1$

• $1 + \tan^2\theta = \sec^2\theta$

• $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

## Ratio identities

• $\tan\theta = \sin\theta / \cos\theta$, when $\cos\theta \neq 0$.

• $\cot\theta = \cos\theta / \sin\theta$, when $\sin\theta \neq 0$.

• $\sec\theta = 1/\cos\theta$; $\operatorname{cosec}\theta = 1/\sin\theta$.

## Worked example

Simplify $(1 - \cos^2\theta)/\sin^2\theta$. Since $1 - \cos^2\theta = \sin^2\theta$, the expression equals 1, wherever the original denominator is nonzero.

## Proof strategy

• Convert sec, cosec, tan, and cot into sine and cosine where useful.

• Use $\sin^2\theta + \cos^2\theta = 1$ to replace one squared ratio.

• Simplify one side of an identity rather than manipulating both sides randomly.

## Practice

• Simplify $(\sec^2\theta - 1)/\tan^2\theta$.

• Show that $(1 + \tan^2\theta)\cos^2\theta = 1$.

• Simplify $(1 - \sin^2\theta)/\cos^2\theta$.

# 5. Compound and Multiple Angles

## Addition and subtraction

• sin(A ± B) = sin A cos B ± cos A sin B

• cos(A ± B) = cos A cos B ■ sin A sin B

• tan(A ± B) = (tan A ± tan B)/(1 ■ tan A tan B), where defined.

## Double-angle formulas

• sin 2A = 2 sin A cos A

• cos 2A = $\cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

• tan 2A = $2\tan A/(1 - \tan^2 A)$, where defined.

## Triple-angle formulas

• sin 3A = $3\sin A - 4\sin^3 A$

• cos 3A = $4\cos^3 A - 3\cos A$

## Worked example

sin 75° = sin(45° + 30°) = (√2/2)(√3/2) + (√2/2)(1/2) = (√6 + √2)/4.

## Practice

• Find cos 15°.

• If sin A = 3/5 and A is acute, find sin 2A.

• Write cos 2A in terms of sin A.

# 6. Transformations and Product Formulas

## Sum-to-product

• $\sin A + \sin B = 2 \sin((A+B)/2) \cos((A-B)/2)$

• $\sin A - \sin B = 2 \cos((A+B)/2) \sin((A-B)/2)$

• $\cos A + \cos B = 2 \cos((A+B)/2) \cos((A-B)/2)$

• $\cos A - \cos B = -2 \sin((A+B)/2) \sin((A-B)/2)$

## Product-to-sum

• $2\sin A \cos B = \sin(A+B) + \sin(A-B)$

• $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

• $2\sin A \sin B = \cos(A-B) - \cos(A+B)$

## When useful

These formulas are useful for simplifying expressions, proving identities, and converting sums into products that are easier to solve.

## Worked example

$\sin 70° + \sin 50° = 2\sin 60° \cos 10° = \sqrt{3} \cos 10°$.

## Practice

• Convert $\cos 5x + \cos 3x$ into a product.

• Express $2\sin 4x \cos 2x$ as a sum.

• Simplify $\sin 5x - \sin x$.

# 7. Trigonometric Equations

## Basic solutions

• $\sin x = 0 \Rightarrow x = n\pi$.

• $\cos x = 0 \Rightarrow x = \pi/2 + n\pi$.

• $\tan x = 0 \Rightarrow x = n\pi$, where n is an integer.

## General patterns

For $\sin x = \sin \alpha$: $x = \alpha + 2n\pi$ or $x = \pi - \alpha + 2n\pi$. For $\cos x = \cos \alpha$: $x = 2n\pi \pm \alpha$. For $\tan x = \tan \alpha$: $x = \alpha + n\pi$.

## Solve within an interval

Always apply the interval restriction after finding the general solution. For $0 \le x < 2\pi$, list only values inside that interval.

## Worked example

Solve $2\sin x = 1$ for $0 \le x < 2\pi$. $\sin x = 1/2$, so $x = \pi/6$ or $5\pi/6$.

## Practice

• Solve $\cos x = -1/2$ for $0 \le x < 2\pi$.

• Solve $\tan x = 1$ for $0 \le x < 2\pi$.

• Solve $2\cos^2 x - 1 = 0$ for $0 \le x < 2\pi$.

# 8. Inverse Trigonometric Functions

## Principal values

Inverse trigonometric functions return a principal angle from a restricted range. For example, arcsin x has range $[-\pi/2, \pi/2]$, arccos x has range $[0, \pi]$, and arctan x has range $(-\pi/2, \pi/2)$.

## Key relationships

- $\sin(\arcsin x) = x$ for $-1 \le x \le 1$.
- $\cos(\arccos x) = x$ for $-1 \le x \le 1$.
- $\tan(\arctan x) = x$ for all real x.

## Caution

$\arcsin(\sin \theta)$ is not always $\theta$. The result must lie in the principal range of arcsin.

## Worked example

$\arcsin(\sin 5\pi/6) = \arcsin(1/2) = \pi/6$, because $\pi/6$ is the principal value.

## Practice

- Find $\arccos(1/2)$.
- Find $\arctan(1)$.
- Evaluate $\arcsin(\sin(-2\pi/3))$.

# 9. Heights and Distances

## Core ideas

Angle of elevation is measured upward from the horizontal. Angle of depression is measured downward from the horizontal. Draw a right triangle and label known sides before choosing a ratio.

## Useful relationships

• $\tan \theta$ = height / horizontal distance.

• $\sin \theta$ = height / line-of-sight distance.

• $\cos \theta$ = horizontal distance / line-of-sight distance.

## Worked example

A point is 20 m from the foot of a tower. If the angle of elevation to the top is 45°, height = 20 tan 45° = 20 m, assuming level ground.

## Problem-solving checklist

• Draw the diagram.

• Identify the right angle.

• Choose sine, cosine, or tangent based on known and required sides.

• Check units and whether the answer is reasonable.

## Practice

• A pole casts a 12 m shadow when the sun's elevation is 30°. Find its height.

• A kite string is 50 m long and makes 30° with the horizontal. Find its height above the hand, ignoring hand height.

• A tower is observed from 30 m away at 60°. Find its height.

# 10. Trigonometry Formula Revision

## Must-know formulas

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $\sin 2\theta = 2\sin\theta\cos\theta$
- $\cos 2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \blacksquare \sin A \sin B$

## Common pitfalls

- Check quadrant signs before using standard values.
- Do not mix degrees and radians in a single calculation.
- Check denominator restrictions when cancelling expressions.
- For inverse functions, respect principal-value ranges.

## Mini test

- Convert 210° to radians.
- Evaluate sin 15°.
- Simplify $(\sec^2 x - 1)$.
- Solve $\sin x = \sqrt{3}/2$ on $[0, 2\pi)$.
- If $\tan\theta = 3/4$ and $\theta$ is acute, find $\sin\theta$.

# 11. Sets, Intervals and Real Numbers

## Set notation

A set is a well-defined collection. Symbols: $\in$ belongs to, $\notin$ does not belong to, $\subset$ is a subset of, $\varnothing$ is the empty set, and U is the universal set.

## Operations

• Union $A \cup B$: elements in A or B or both.

• Intersection $A \cap B$: elements common to both.

• Difference $A - B$: elements in A but not B.

• Complement $A'$: elements in U that are not in A.

## Intervals

• (a,b): endpoints excluded.

• [a,b]: endpoints included.

• [a,b): a included, b excluded.

• Use interval notation when writing inequality solutions.

## Worked example

If $A = \{1,2,3,4\}$ and $B = \{3,4,5\}$, then $A \cup B = \{1,2,3,4,5\}$, $A \cap B = \{3,4\}$, and $A - B = \{1,2\}$.

## Practice

• If $U = \{1,2,3,4,5,6\}$, $A = \{2,4,6\}$, find $A'$.

• Write $-2 < x \leq 5$ in interval notation.

• How many subsets does a set with 4 elements have?

# 12. Indices, Surds and Logarithms

## Laws of indices

• a■ × a■ = a■■■

• a■/a■ = a■■■ for a ≠ 0

• (a■)■ = a■■

• a■ = 1 for a ≠ 0

• a■■ = 1/a■

## Surds

Simplify radicals by extracting perfect-square factors. Example: √72 = √(36×2) = 6√2.

## Logarithm laws

• log■(xy) = log■x + log■y

• log■(x/y) = log■x – log■y

• log■(x■) = n log■x

• log■x = log_bx / log_ba

## Conditions

For real logarithms, base a > 0, a ≠ 1, and argument x > 0.

## Practice

• Simplify $2^{3}$ × 2■ / 2■.

• Simplify √108.

• Expand log■(9$x^{2}$/y).

# 13. Polynomials and Factorisation

## Polynomial basics

The degree is the highest exponent with a non-zero coefficient. A root $\alpha$ satisfies $p(\alpha) = 0$.

## Useful identities

- $a^2 - b^2 = (a-b)(a+b)$
- $a^2 + 2ab + b^2 = (a+b)^2$
- $a^2 - 2ab + b^2 = (a-b)^2$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

## Remainder and factor theorem

Remainder on division by $(x-a)$ is $p(a)$. If $p(a)=0$, then $(x-a)$ is a factor of $p(x)$.

## Worked example

Factor $x^2 - 5x + 6$. Find two numbers whose product is 6 and sum is $-5$: $-2$ and $-3$. Thus $x^2 - 5x + 6 = (x-2)(x-3)$.

## Practice

- Factor $x^2 - 9$.
- Find the remainder when $x^3 - 2x + 1$ is divided by $x-2$.
- Factor $x^3 - 8$.

# 14. Quadratic Equations

## Standard form

$ax^2 + bx + c = 0$, with $a \neq 0$.

## Quadratic formula

$x = [-b \pm \sqrt{(b^2 - 4ac)}]/(2a)$

## Discriminant

- $D = b^2 - 4ac > 0$: two distinct real roots.
- $D = 0$: equal real roots.
- $D < 0$: no real roots (two complex roots).

## Sum and product of roots

If roots are $\alpha$ and $\beta$, then $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$.

## Worked example

Solve $x^2 - 5x + 6 = 0$. Factorise: $(x-2)(x-3)=0$, so $x = 2$ or $x = 3$.

## Practice

- Solve $2x^2 - 7x + 3 = 0$.
- Find the nature of roots of $3x^2 + 2x + 4 = 0$.
- Form a monic quadratic with roots 4 and $-1$.

# 15. Inequalities and Modulus

## Linear inequalities

Solve like equations, but reverse the inequality sign whenever multiplying or dividing by a negative number.

## Modulus meaning

|x| is the distance of x from zero. Therefore $|x| \geq 0$.

## Standard results

- $|x| < a$ means $-a < x < a$, for $a > 0$.
- $|x| \leq a$ means $-a \leq x \leq a$.
- $|x| > a$ means $x < -a$ or $x > a$.
- $|x-c|$ represents distance from c.

## Worked example

Solve $|x-2| < 3$. Then $-3 < x-2 < 3$, so $-1 < x < 5$.

## Practice

- Solve $3x - 4 \leq 8$.
- Solve $|2x+1| = 5$.
- Solve $|x| \geq 4$.

# 16. Permutations and Combinations

## Factorial

n! = n(n–1)(n–2)…2·1 and 0! = 1.

## Permutations

Arrangements where order matters: nP■ = n!/(n–r)!.

## Combinations

Selections where order does not matter: nC■ = n!/[r!(n–r)!].

## Relationship

nP■ = nC■ × r!. Also nC■ = nC■■■.

## Worked example

From 5 people, choose 2: 5C2 = 5!/(2!3!) = 10. Arrange 2 from 5: 5P2 = 5×4 = 20.

## Practice

• Evaluate 6P3.

• Evaluate 8C2.

• How many different arrangements are possible for the letters A, B, C, D?

# 17. Binomial Theorem

## Expansion

(a+b)■ = Σ■■■■ nC■ a■■■ b■.

## General term

T■■■ = nC■ a■■■ b■.

## Properties

• Coefficients in (a+b)■ are nC0, nC1, …, nCn.

• The sum of coefficients is 2■.

• For (a–b)■, signs alternate according to the power of b.

## Worked example

$(x+2)^3 = x^3 + 3x^2(2) + 3x(4) + 8 = x^3 + 6x^2 + 12x + 8$.

## Practice

• Expand (x–1)■.

• Find the coefficient of $x^2$ in (x+3)■.

• Find the sum of coefficients in (2x–1)■ by setting x = 1.

# 18. Complex Numbers

## Definition

A complex number is $z = a + ib$, where $i^2 = -1$ and $a,b$ are real. Real part is $a$ and imaginary part is $b$.

## Operations

- $(a+ib)+(c+id) = (a+c)+i(b+d)$.
- $(a+ib)(c+id) = (ac-bd)+i(ad+bc)$.
- Conjugate of $a+ib$ is $a-ib$.
- $|a+ib| = \sqrt{(a^2+b^2)}$.

## Division

Multiply numerator and denominator by the conjugate of the denominator.

## Worked example

$(2+3i)(1-i) = 2-2i+3i-3i^2 = 5+i$.

## Practice

- Find the conjugate of $4-7i$.
- Compute $(3+i)/(1-i)$.
- Find the modulus of $5-12i$.

# 19. Matrices and Determinants

## Matrix basics

A matrix is a rectangular array. Its order is rows × columns. Addition requires equal orders; multiplication AB is defined when columns of A equal rows of B.

## 2×2 determinant

For A = [[a,b],[c,d]], det(A) = ad − bc.

## Inverse of a 2×2 matrix

If ad−bc ≠ 0, A■¹ = 1/(ad−bc) [[d,−b],[−c,a]].

## Worked example

For A = [[2,1],[3,4]], det(A) = 2×4 − 1×3 = 5.

## Practice

• Find the determinant of [[1,2],[5,3]].

• State when a 2×2 matrix is singular.

• Find A+B for A=[[1,2],[3,4]] and B=[[0,1],[2,0]].

# 20. Sequences and Series

## Arithmetic progression (AP)

• nth term: a■ = a + (n–1)d.

• Sum: S■ = n/2[2a + (n–1)d] = n(a + l)/2.

## Geometric progression (GP)

• nth term: a■ = ar■■¹.

• Sum for r ≠ 1: S■ = a(r■–1)/(r–1), equivalently a(1–r■)/(1–r).

• Infinite sum for |r| < 1: S∞ = a/(1–r).

## Worked example

AP: 3, 7, 11, … has a = 3, d = 4. The 10th term is 3 + 9×4 = 39.

## Practice

• Find the 20th term of 5, 8, 11, ….

• Find the sum of the first 10 positive integers.

• Find the infinite sum 8 + 4 + 2 + …

# 21. Algebra: Mixed Review

## Formula bank

• Quadratic roots: $(-b \pm \sqrt{(b^2-4ac)})/(2a)$.

• nP■ = n!/(n–r)!; nC■ = n!/[r!(n–r)!].

• AP nth term: a+(n–1)d; GP nth term: ar■■¹.

• log■(xy)=log■x+log■y; a■a■=a■■■.

## Mixed practice

• Factor $x^2 + x - 12$.

• Solve $2x^2 + x - 6 = 0$.

• Evaluate 7C2.

• Find the 12th term of an AP with a=4, d=3.

• Simplify log■ 32.

• Find the modulus of 6+8i.

## Self-check method

For each missed question, write the correct rule beside the mistake and solve a similar question immediately.

# 22. Functions, Limits and Continuity

## Functions

A function assigns exactly one output to each input in its domain. Know the domain, range, zeros, and points where the expression is undefined.

## Limit idea

lim■→■ f(x) is the value approached by f(x) as x approaches a. The function value at a may differ from the limit.

## Basic limit laws

• lim(f+g)=lim f + lim g.

• lim(fg)=(lim f)(lim g).

• lim(f/g)=(lim f)/(lim g), when the denominator limit is non-zero.

## Continuity

A function is continuous at x=a if f(a) exists, the limit as x→a exists, and lim■→■ f(x)=f(a).

## Worked example

lim■→■ (x²–4)/(x–2) = lim■→■ (x–2)(x+2)/(x–2) = 4.

## Practice

• Evaluate lim■→■ (x²–9)/(x–3).

• Find the domain of 1/(x–5).

• State the three conditions for continuity at a point.

# 23. Standard Limits

## Important limits (radians)

• lim■→■ sin x/x = 1.

• lim■→■ (1–cos x)/x² = 1/2.

• lim■→■ tan x/x = 1.

• lim■→■ (e■–1)/x = 1.

• lim■→■ ln(1+x)/x = 1.

## Algebraic technique

For 0/0 forms, try factorisation, rationalisation, or a standard limit. Do not substitute into an indeterminate form and stop.

## Worked example

lim■→■ sin 5x/x = 5 × lim■→■ sin 5x/(5x) = 5.

## Rationalisation example

lim■→■ [√(1+x)–1]/x. Multiply top and bottom by √(1+x)+1 to obtain 1/[√(1+x)+1] → 1/2.

## Practice

• Evaluate lim■→■ sin 7x/x.

• Evaluate lim■→■ (1–cos 3x)/x².

• Evaluate lim■→■ (√(4+x)–2)/x.

# 24. Differentiation: First Principles

## Definition

f′(x) = lim■→■ [f(x+h)–f(x)]/h, if the limit exists.

## Meaning

The derivative is the instantaneous rate of change and the slope of the tangent to the curve at a point.

## Worked example: f(x)=x²

f(x+h)=(x+h)²=x²+2xh+h². Difference quotient = (2xh+h²)/h = 2x+h. Let h→0: f′(x)=2x.

## Derivative notation

Common forms are f′(x), dy/dx, and D■[y].

## Practice

• Use first principles to differentiate f(x)=3x+2.

• Use first principles to differentiate x²+1.

• Explain the geometric meaning of f′(a).

# 25. Basic Differentiation Rules

## Core rules

• d/dx(c)=0.

• d/dx(x■)=nx■■¹.

• d/dx[cf]=c f′.

• d/dx(f±g)=f′±g′.

• Product: (fg)′ = f′g + fg′.

• Quotient: (f/g)′ = (f′g–fg′)/g², g≠0.

## Chain rule

If y=f(g(x)), then dy/dx = f′(g(x))g′(x). Differentiate the outer function first, then multiply by the derivative of the inner function.

## Worked examples

• d/dx(5x■–3x²+7)=20x³–6x.

• d/dx[(x²+1)(x³)] = 2x·x³ + (x²+1)3x² = 5x■+3x².

• d/dx(3x+1)■ = 15(3x+1)■.

## Practice

• Differentiate 4x■ – 2x + 9.

• Differentiate x² sin x.

• Differentiate (2x²+1)³.

# 26. Derivatives of Trigonometric and Exponential Functions

## Standard derivatives

• d/dx(sin x)=cos x.

• d/dx(cos x)=−sin x.

• d/dx(tan x)=sec²x.

• d/dx(cot x)=−cosec²x.

• d/dx(sec x)=sec x tan x.

• d/dx(cosec x)=−cosec x cot x.

• d/dx(e■)=e■.

• d/dx(a■)=a■ ln a.

• d/dx(ln x)=1/x, for x>0.

## Chain rule examples

• d/dx sin(3x)=3cos(3x).

• d/dx e^(2x)=2e^(2x).

• d/dx ln(5x+1)=5/(5x+1).

## Important note

The standard trigonometric derivative formulas assume the angle is measured in radians.

## Practice

• Differentiate cos 4x.

• Differentiate tan(x²).

• Differentiate ln(x²+1).

# 27. Implicit and Logarithmic Differentiation

## Implicit differentiation

When x and y appear together, differentiate both sides with respect to x. Whenever differentiating a term involving y, multiply by dy/dx.

## Worked example

$x^2+y^2=25$. Differentiating gives $2x+2y(dy/dx)=0$, so $dy/dx=-x/y$ when $y\neq0$.

## Logarithmic differentiation

Useful when variables occur in both base and exponent or when a product has many factors. Take ln of both sides, then differentiate.

## Example: y=x■

ln y = x ln x. Differentiate: y′/y = ln x + 1. Therefore y′ = x■(ln x+1), for x>0.

## Practice

• Find dy/dx if $xy + y^2 = 4$.

• Differentiate $y=(x^2+1)$■ using logarithmic differentiation.

• Differentiate y=x^(2x), for x>0.

# 28. Applications of Derivatives

## Tangent and normal

At $x=a$, tangent slope is $m=f'(a)$. Tangent equation: $y-f(a)=f'(a)(x-a)$. The normal slope is $-1/f'(a)$ when $f'(a)\neq 0$.

## Increasing and decreasing

- If $f'(x)>0$ on an interval, f is increasing there.
- If $f'(x)<0$ on an interval, f is decreasing there.
- Critical points occur where $f'(x)=0$ or the derivative is undefined.

## Maxima and minima

A stationary point has $f'(x)=0$. A sign change from + to – indicates a local maximum; from – to + indicates a local minimum.

## Worked example

For $f(x)=x^2-4x+1$, $f'(x)=2x-4=0$ at $x=2$. Since $f'$ changes from negative to positive, $x=2$ is a minimum; $f(2)=-3$.

## Practice

- Find the tangent to $y=x^2$ at $x=1$.
- Find the critical point of $x^3-3x$.
- Find intervals where $x^2-6x$ is decreasing.

# 29. Integration: Antiderivatives

## Meaning

Integration reverses differentiation. If $F'(x)=f(x)$, then $\int f(x)dx=F(x)+C$, where C is the constant of integration.

## Power rule

$\int x^n\,dx = x^{n+1}/(n+1)+C$, for $n\neq -1$.

## Basic integrals

- $\int k\,dx = kx+C$.
- $\int 1/x\,dx = \ln|x|+C$.
- $\int e^x\,dx = e^x+C$.
- $\int \cos x\,dx = \sin x+C$.
- $\int \sin x\,dx = -\cos x+C$.
- $\int \sec^2 x\,dx = \tan x+C$.

## Worked example

$\int(3x^2-4x+2)dx = x^3-2x^2+2x+C$.

## Practice

- Integrate $5x^n + 2x$.
- Integrate $1/x$.
- Integrate $\cos 3x$.

# 30. Integration Techniques

## Substitution

If an integrand contains a function and its derivative, set u equal to the inner function. Then du = u′(x)dx and rewrite the integral in u.

## Example

∫2x(x²+1)³ dx. Let u=x²+1, du=2x dx. Integral = ∫u³du = u■/4+C = (x²+1)■/4+C.

## Integration by parts

∫u dv = uv – ∫v du. Choose u so that its derivative becomes simpler.

## Example

∫x e■ dx = x e■ – ∫e■ dx = e■(x–1)+C.

## Practice

• Integrate 3x²(x³+2)■.

• Integrate x cos x using parts.

• Integrate e^(2x).

# 31. Definite Integrals and Area

## Fundamental theorem

If $F'(x)=f(x)$, then $\int_a^b f(x)dx = F(b)-F(a)$.

## Properties

- $\int_a^a f(x)dx = 0$.
- $\int_a^b f(x)dx = -\int_b^a f(x)dx$.
- $\int_a^b f + \int_b^c f = \int_a^c f$.
- For an even function: $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$.
- For an odd function: $\int_{-a}^{a} f(x)dx = 0$.

## Worked example

$\int_1^2 3x^2\,dx = [x^3]_1^2 = 8$.

## Area

A definite integral gives signed area. Where a curve lies below the x-axis, its integral is negative; geometric area is found by splitting intervals or using absolute values.

## Practice

- Evaluate $\int_0^3 2x\,dx$.
- Evaluate $\int_0^{\pi} \sin x\,dx$.
- State the integral of an odd function over $[-a,a]$.

# 32. Differential Equations: Basics

## Definition

A differential equation contains an unknown function and one or more of its derivatives. The order is the highest derivative present.

## Formation and solution

A general solution contains arbitrary constants. Apply initial conditions to determine those constants.

## Separable equations

If dy/dx = g(x)h(y), rearrange to dy/h(y)=g(x)dx and integrate both sides.

## Worked example

dy/dx = 2x. Integrating gives $y=x^2+C$. If y(0)=3, then C=3 and $y=x^2+3$.

## Practice

- Find the general solution of $dy/dx=3x^2$.
- Solve dy/dx = y, stating the general solution.
- Find C if $y=x^2+C$ and y(2)=7.

# 33. Calculus Formula Revision

## Differentiation

- d/dx(x■)=nx■■¹.
- d/dx(sin x)=cos x.
- d/dx(cos x)=−sin x.
- d/dx(e■)=e■.
- d/dx(ln x)=1/x.
- (fg)′=f′g+fg′.
- Chain rule: d/dx f(g(x))=f′(g(x))g′(x).

## Integration

- ∫x■dx=x■■¹/(n+1)+C, n≠−1.
- ∫1/x dx=ln|x|+C.
- ∫e■dx=e■+C.
- ∫cos x dx=sin x+C.
- ∫sin x dx=−cos x+C.
- ∫■■ f(x)dx=F(b)−F(a).

## Before the exam

- Check whether a question asks for derivative, antiderivative, or definite integral.
- Remember the constant C for indefinite integrals.
- Use radians for standard trigonometric limits and derivatives.
- Check the domain before using logarithms or cancelling factors.

# 34. NDA-Style Mixed Practice Set A

## Instructions

Attempt these without notes. Suggested time: 25 minutes. Questions are original practice questions, not copied from an official paper.

## Questions 1–8

- 1. Convert 135° to radians.
- 2. Evaluate sin 30° + cos 60°.
- 3. Simplify $(1-\cos^2 x)/\sin^2 x$.
- 4. Factor $x^2-7x+12$.
- 5. Solve $x^2-9=0$.
- 6. Evaluate 6C2.
- 7. Find the 8th term of the AP 2, 5, 8, ….
- 8. Simplify log■■(1000).

## Questions 9–12

- 9. Evaluate lim■→■ sin 4x/x.
- 10. Differentiate 3x■–2x².
- 11. Differentiate sin(5x).
- 12. Integrate $6x^2\,dx$.

## Answer check

1) $3\pi/4$; 2) 1; 3) 1; 4) (x–3)(x–4); 5) ±3; 6) 15; 7) 23; 8) 3; 9) 4; 10) $12x^3-4x$; 11) $5\cos(5x)$; 12) $2x^3+C$.

# 35. NDA-Style Mixed Practice Set B

## Questions 13–20

- 13. Find cos 150°.
- 14. If tan θ=3/4 and θ is acute, find sin θ.
- 15. Solve $2x^2-5x-3=0$.
- 16. Find the discriminant of $x^2+4x+4=0$.
- 17. Expand $(x+2)^3$.
- 18. Find the modulus of $3-4i$.
- 19. Differentiate $x^2e^{\blacksquare}$.
- 20. Integrate $(2x+1)\,dx$.

## Questions 21–26

- 21. Solve $\sin x=0$ for $0\le x<2\pi$.
- 22. Evaluate $\int_{\blacksquare}^{1} 2x\,dx$.
- 23. Find the tangent slope to $y=x^3$ at $x=2$.
- 24. Evaluate $\lim_{\blacksquare\to\blacksquare} (x^2-4)/(x-2)$.
- 25. Find the 5th term of a GP with $a=3$ and $r=2$.
- 26. Solve $|x-1|=4$.

## Answers

13) $-\sqrt{3}/2$; 14) 3/5; 15) x=3 or −1/2; 16) 0; 17) $x^3+6x^2+12x+8$; 18) 5; 19) $e^{\blacksquare}(x^2+2x)$; 20) $x^2+x+C$; 21) 0, π; 22) 1; 23) 12; 24) 4; 25) 48; 26) x=5 or −3.

# 36. Final Formula Sheet and Study Checklist

## Trigonometry essentials

• 180°=π radians; s=rθ; sector area=½r²θ.

• sin²x+cos²x=1; 1+tan²x=sec²x.

• sin 2x=2sin x cos x; cos 2x=cos²x–sin²x.

• sin(A±B)=sin A cos B±cos A sin B.

## Algebra essentials

• a²–b²=(a–b)(a+b).

• Quadratic formula: x=(–b±√(b²–4ac))/(2a).

• nP■=n!/(n–r)!; nC■=n!/[r!(n–r)!].

• AP: a■=a+(n–1)d; GP: a■=ar■■¹.

## Calculus essentials

• lim■→■ sin x/x=1 (x in radians).

• d/dx(x■)=nx■■¹; d/dx(e■)=e■; d/dx(ln x)=1/x.

• ∫x■dx=x■■¹/(n+1)+C, n≠–1.

• ∫■■ f(x)dx=F(b)–F(a).

## Final checklist

• I can recall standard trig values without a table.

• I can factor and solve quadratic equations confidently.

• I can apply product, quotient, and chain rules.

• I can evaluate basic indefinite and definite integrals.

• I have reviewed every error from my practice sets.

## Keep going

Consistency beats last-minute cramming. Revise, practise, analyse mistakes, and repeat. — Defence Dreams